LOCAL D^0 HADRONIC MATRIX ELEMENTS FROM 2+1 LATTICE QCD

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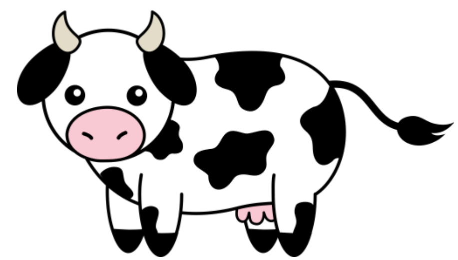


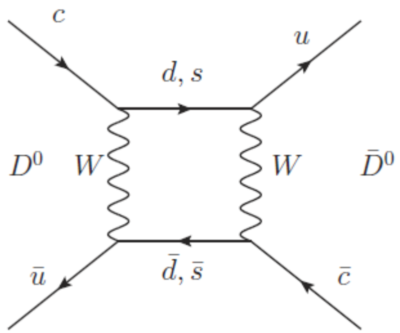
Fermilab

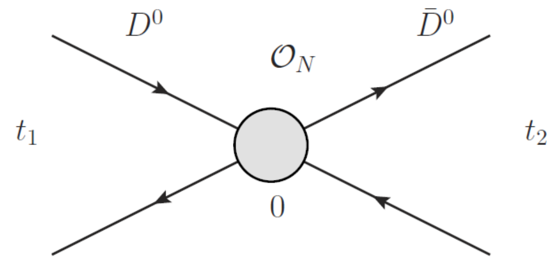
Outline

- Background/Motivation
- D Mixing Overview
- Lattice Formulation
- Analysis Overview
- Preliminary Results

Lattice QCD's role in flavor physics

For a general physical process involving hadrons only part of the diagram may be solved perturbatively.





$$\left\langle \bar{D}^0 \left| H \right| D^0 \right\rangle \to C \left\langle \bar{D}^0 \left| \mathcal{O} \right| D^0 \right\rangle$$

- Our aim is to integrate the path integral numerically.

$$\langle \Omega \, | A | \, \Omega \rangle = \frac{\int [d\psi][d\bar{\psi}][dU] A[\psi, \bar{\psi}, U] e^{-S_U - \bar{\psi}(\not{D}+m)\psi}}{\int [d\psi][d\bar{\psi}][dU] e^{-S_U - \bar{\psi}(\not{D}+m)\psi}}$$

Weighted average

- Use Monte Carlo to generate field configurations w/ distribution

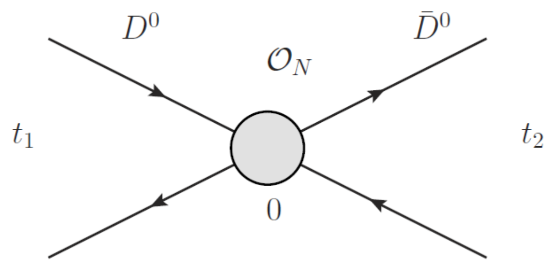
$$e^{-S_U + \ln\det(\not{D}+m)} \Rightarrow \langle \Omega \, | A | \, \Omega \rangle \simeq \frac{1}{N} \sum_{n=1}^{N} A(U_n)$$

Simple average

Background (0)

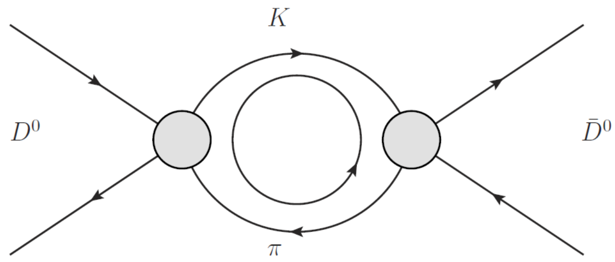
D Mixing occurs via two types of diagrams

Short distance



I am looking at this diagram

Long distance



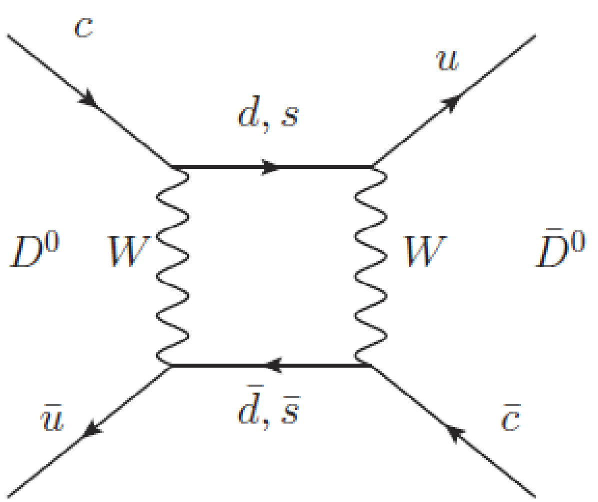
This is much harder to understand

Background (1)

Phenomelogically very different from Kaon and B mixing

Short Distance

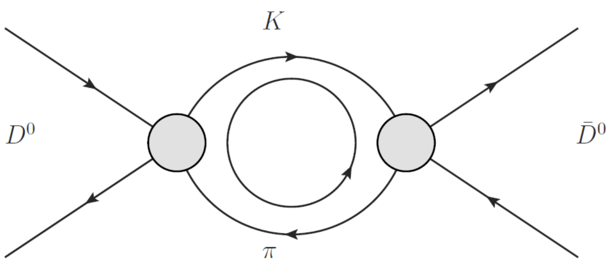
- Strange and down are GIM suppressed.
- Bottom is $|V_{ub}V^*_{cb}|^2$ suppressed. Described by two flavors in the SM. This implies that D mixing has no CP violation in the SM.
- The contribution of SM short distance to D mixing is expected to be small due to these severe suppressions.



- HFAG 2013: $x = (0.419 \pm 0.211)\%$
  $y = (0.456 \pm 0.186)\%$ CLEO, Belle, BaBar

- SM short distance estimate: $x \sim y \sim 10^{-4}\%$ Golowich hep-ph/0506185v1 w/ Gupta quenched lattice

Background (2)



Long Distance

- Long distance effects of D mixing are not well understood.
- Phenomenology vs. long distance diagrams:
  - Inclusive approach: Heavy quark expansion around Λ_{QCD}/m_c
  - Exclusive approach: Many intermediate states. Large hadronic uncertainties.
- Lattice vs. long distance diagrams:
  - Disconnected diagrams
  - Many contributions from different intermediate states
  - Multi-particle intermediate states
- Order of magnitude estimates suggest that these diagrams enhance the SM mixing amplitude.

Motivation (1)

Phenomelogical Implications

- Possibility of observing New Physics enhancements to the short distance diagram via New Physics (NP).
- The charm community is interested in having unquenched extractions of short distance D mixing matrix elements in order to make NP predictions.
- LHCb, CDF and Belle observe CPV through D decays (2012). However recent LHCb analysis (2013) for D decays produced from semileptonic B decays have not confirmed this observation. Active field of research in experimental high energy physics. Era of high precision measurements of charm physics!

Motivation (2)

Mature Lattice Formulation

- D mixing is a “gold plated” process.
  - Has one initial and final bound state particle connected by a short distance interaction
  - Theoretically well understood in lattice
- Have competitors. ETMC is currently publishing a paper on D mixing as well. (Vela, Lattice 2013)

This project is part of a larger continual effort of the Fermilab/MILC collaboration’s contribution to the understanding of flavor physics.

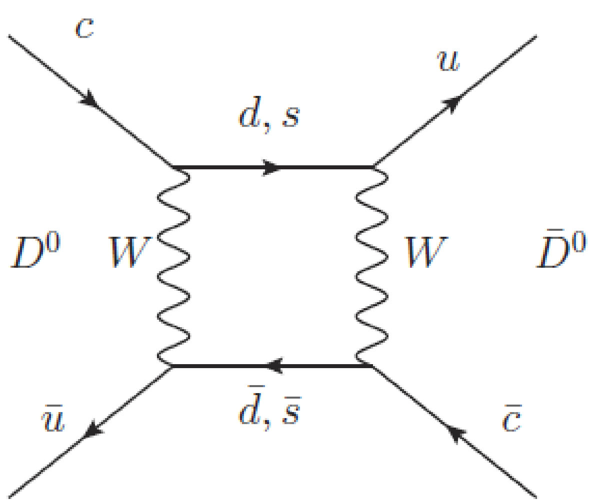
Next generation B factories are coming online. Having a **first principle** determination of the D mixing matrix elements will contribute to our understanding in flavor physics.

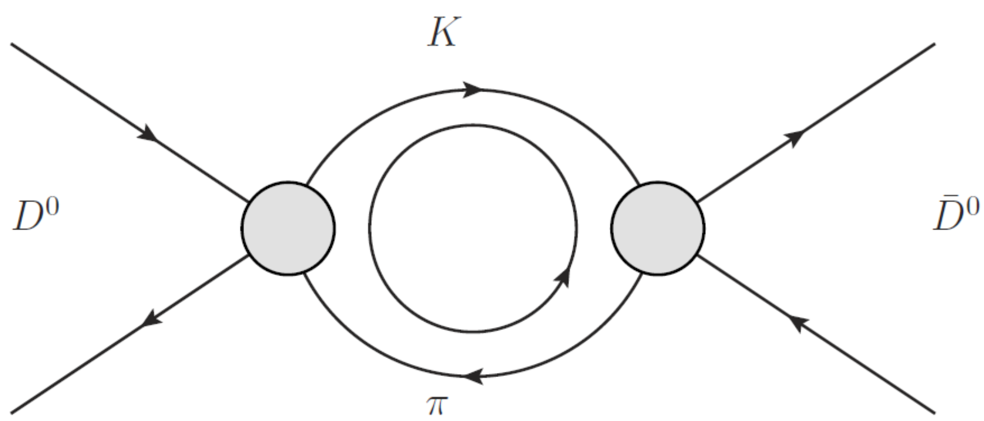
D Mixing Overview

Off-diagonal element of the effective mixing Hamiltonian is:

$$M_{12} - \frac{i}{2}\Gamma_{12} = \sum_i C_i^{(2)} \left\langle \bar{D}^0 \left| \mathcal{O}_i^{\Delta_c=2} \right| D^0 \right\rangle$$

$$+ \sum_{f;jk} \frac{C_j^{(1)} C_k^{(1)} \left\langle \bar{D}^0 \left| \mathcal{O}_j^{\Delta_c=1} \right| f \right\rangle \left\langle f \left| \mathcal{O}_k^{\Delta_c=1} \right| D^0 \right\rangle}{E_f - M_{D^0} + i\epsilon}$$





Matrix Elements

4-quark operators ($\Delta c = 2$) terms:
opts:

$$\mathcal{O}_1 = \bar{c}^\alpha \gamma^\mu \hat{L} u^\alpha \bar{c}^\beta \gamma^\mu \hat{L} u^\beta$$

$$\mathcal{O}_2 = \bar{c}^\alpha \hat{L} u^\alpha \bar{c}^\beta \hat{L} u^\beta$$

$$\mathcal{O}_3 = \bar{c}^\alpha \hat{L} u^\beta \bar{c}^\beta \hat{L} u^\alpha$$

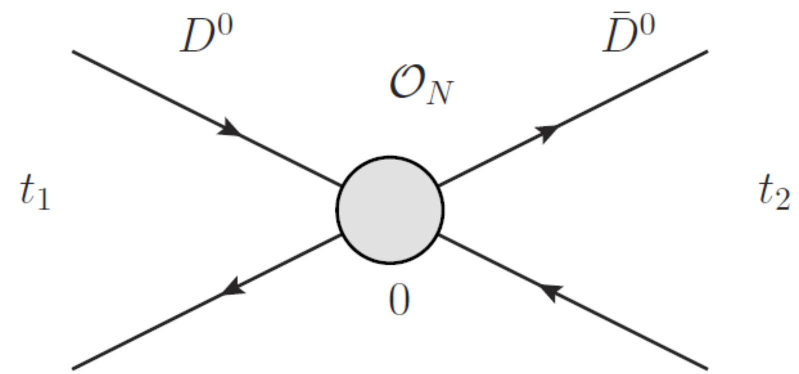
$$\mathcal{O}_4 = \bar{c}^\alpha \hat{L} u^\alpha \bar{c}^\beta \bar{R} u^\beta$$

$$\mathcal{O}_5 = \bar{c}^\alpha \hat{L} u^\beta \bar{c}^\beta \hat{R} u^\alpha$$

D meson creation

$$\chi_{D^0}(x) = \bar{u}\gamma_5 c(x)$$

$$\chi_{\bar{D}^0}(x) = \bar{c}\gamma_5 u(x)$$



Correlation functions

$$C^{2pt}(x,0) = \langle T\{\bar{\chi}^0(x)\,\chi^0(0)\}\rangle$$

$$C_N^{3pt}(x_1,x_2,0) = \langle T\{\chi^0(x_2)\,\mathcal{O}_N(0)\,\chi^0(x_1)\}\rangle$$

Lattice Overview

- Lattice Actions
  - MILC AsqTad 2+1 gauge configurations
    - Light quark (degenerate up and down plus physical strange sea quarks, valence up quark for D mesons)
  - Fermilab interpretation of the Wilson action.
    - Heavy valence quark (charm quark with physical kinetic mass)
- Gauge field generation
  - Monte Carlo generation of gauge fields and sea quarks.
  - All operators in the lattice action depend only on gauge fields.
- Propagators
  - Invert the fermion operator (this is literally a matrix that is inverted).

Ensemble Overview

SO MUCH & MANY DATA!!!!!

a [fm]	$(\frac{L}{a})^3 \times \frac{T}{a}$	(am_l, am_h)	am_q
0.12	$24^3 \times 64$	(0.005, 0.05)	0.005, 0.007, 0.01, 0.02, 0.03, 0.0349, 0.0415, 0.05
0.12	$20^3 \times 64$	(0.007, 0.05)	0.005, 0.007, 0.01, 0.02, 0.03, 0.0349, 0.0415, 0.05
0.12	$20^3 \times 64$	(0.010, 0.05)	0.005, 0.007, 0.01, 0.02, 0.03, 0.0349, 0.0415, 0.05
0.12	$20^3 \times 64$	(0.020, 0.05)	0.005, 0.007, 0.01, 0.02, 0.03, 0.0349, 0.0415, 0.05
0.09	$64^3 \times 96$	(0.00155, 0.031)	0.00155, 0.0031, 0.0062, 0.0093, 0.0124, 0.0261, 0.031
0.09	$40^3 \times 96$	(0.0031, 0.031)	0.0031, 0.0047, 0.0062, 0.0093, 0.0124, 0.0261, 0.031
0.09	$32^3 \times 96$	(0.00465, 0.031)	0.0031, 0.0047, 0.0062, 0.0093, 0.0124, 0.0261, 0.031
0.09	$28^3 \times 96$	(0.0062, 0.031)	0.0031, 0.0047, 0.0062, 0.0093, 0.0124, 0.0261, 0.031
0.09	$28^3 \times 96$	(0.0124, 0.031)	0.0031, 0.0047, 0.0062, 0.0093, 0.0124, 0.0261, 0.031
0.06	$64^3 \times 144$	(0.0018, 0.018)	0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.016, 0.0188
0.06	$56^3 \times 144$	(0.0018, 0.018)	0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.016, 0.0188
0.06	$48^3 \times 144$	(0.0018, 0.018)	0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.016, 0.0188
0.06	$48^3 \times 144$	(0.0018, 0.018)	0.0018, 0.0025, 0.0036, 0.0054, 0.0072, 0.016, 0.0188
0.045	$64^3 \times 192$	(0.0028, 0.014)	0.0018 0.0028 0.004 0.0056 0.0084 0.013 0.016

- Different lattice spacings a ← continuum extrapolation
- Lattice size $\mathrm{m}_\pi L \sim 4$ ← negligible finite volume effects
- Different light quark masses am_l ← sea quark chiral extrapolation
- Different light valence masses am_q ← valence chiral extrapolation

Fit Functions

The fit functions including periodic boundary conditions to the first order in $e^{-E_n T}$ are:

$$C^{2pt}(t) = \sum_n (-1)^{n(t+1)} \frac{|Z_n|^2}{2E_n} \left(e^{-E_n t} + e^{-E_n (T-t)} \right)$$

$$C^{3pt}(t_2, t_1) = \sum_{m,n} (-1)^{n(t_2+1)} (-1)^{m(|t_1|+1)} \frac{\langle n | \mathcal{O}_i | m \rangle Z_n^\dagger Z_m}{4 E_n E_m} e^{-E_n t_2} e^{-E_m |t_1|} + \mathcal{O}\left(e^{-ET}\right)$$

In the code, the periodic boundary conditions of the 3pt is not included.

- analysis shows that 3pt data did not contain any pbc signal
- it adds extra parameters unconstrained by the two point.

The fit functions require distinction between a sum of exponentials.
Bayesian method is used to guide the fitter towards the physically correct minima.

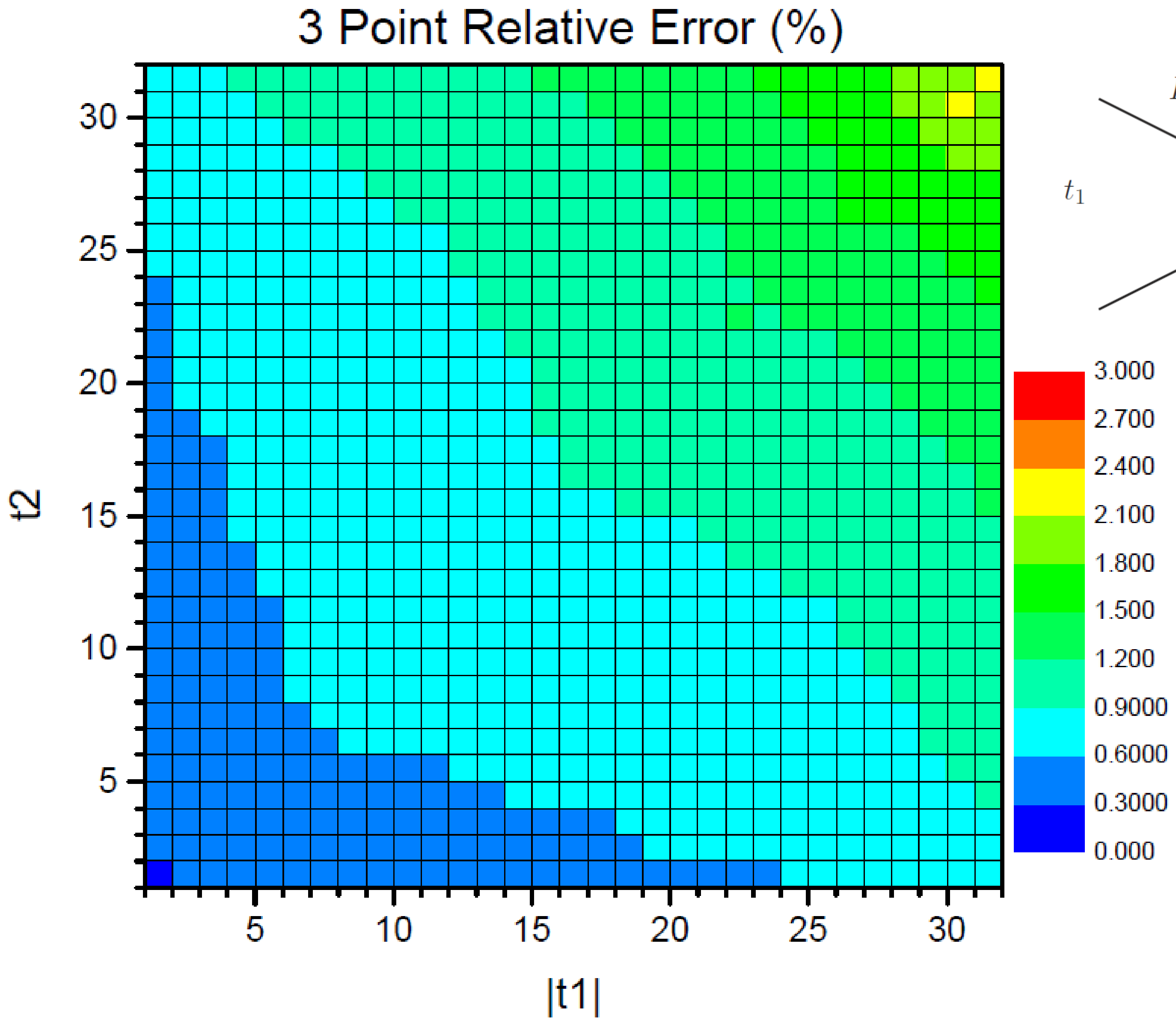
Analysis Overview

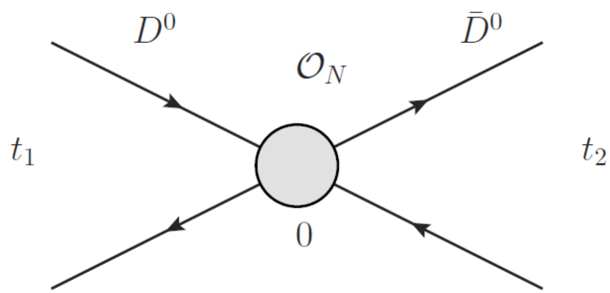
- Data layout
- Stability plots
- Covariance matrices
- Fit ranges

Lattice analysis typically involves a lot of technical details. I want to talk about one that I found to be initially frustrating, but also quite cute in the end.

- In extra slides: Prior choices, Time range choices, Data relative error, P-value distributions, Fit shape discussion

Data Layout

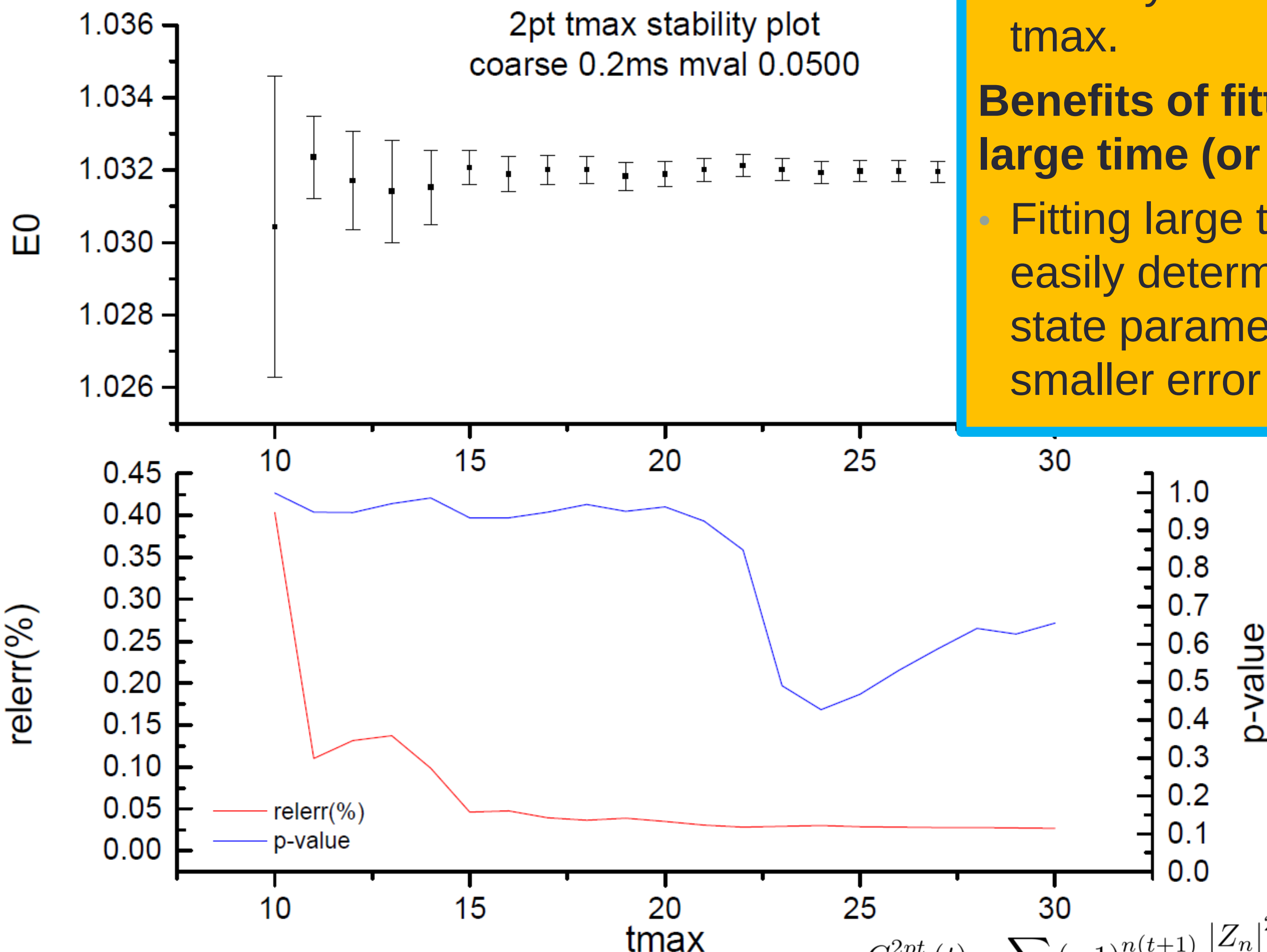




Very small errors!

Let us try to fit over as much data as possible!

2-pt Tmax Stability Plot



What are we looking for?

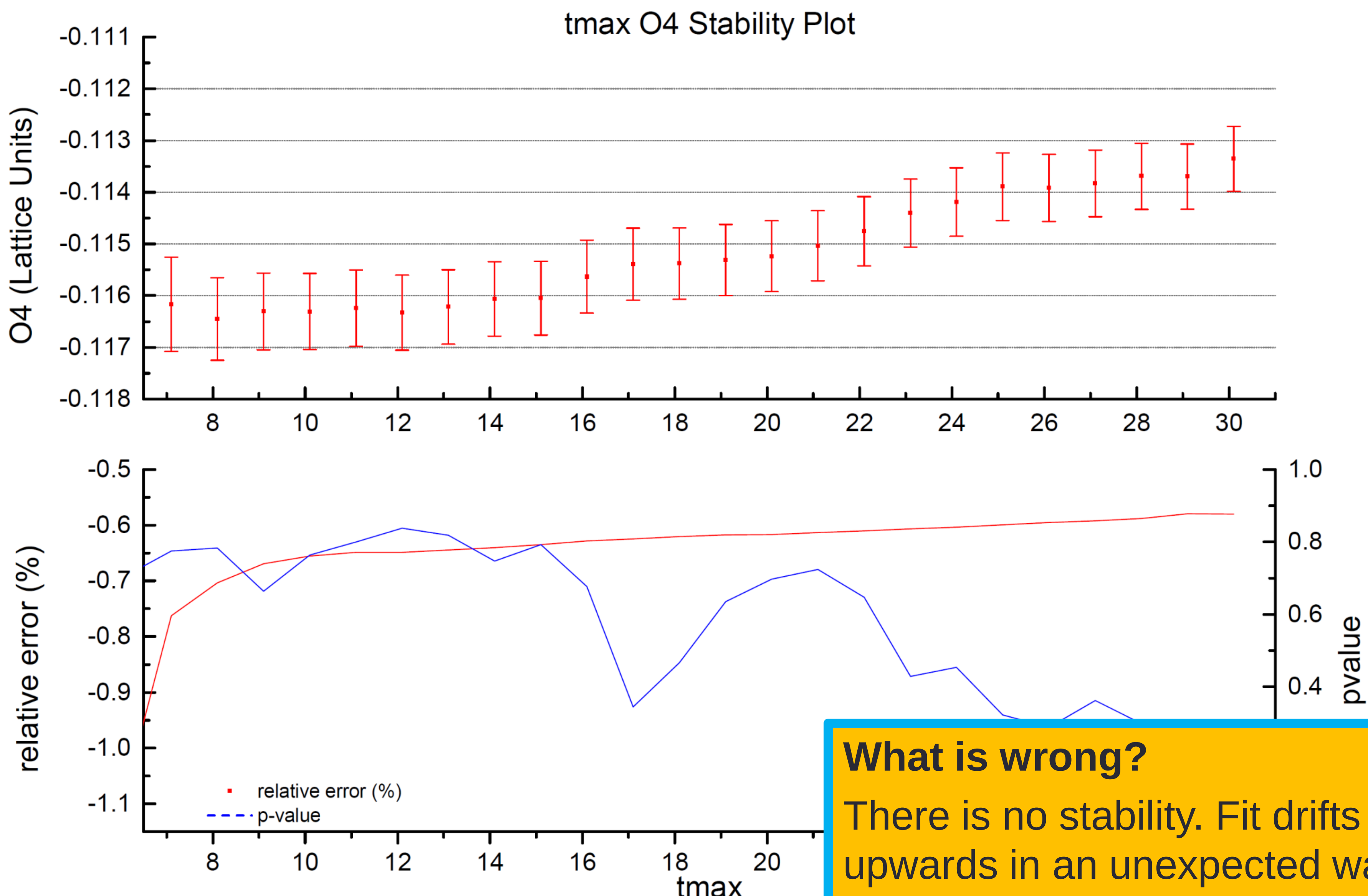
- Stability with increasing tmax.

Benefits of fitting towards large time (or more data)

- Fitting large t allows fitter to easily determine ground state parameters and yield smaller error bars.

$$C^{2pt}(t) = \sum_n (-1)^{n(t+1)} \frac{|Z_n|^2}{2E_n} \left(e^{-E_n t} + e^{-E_n (T-t)}\right)$$

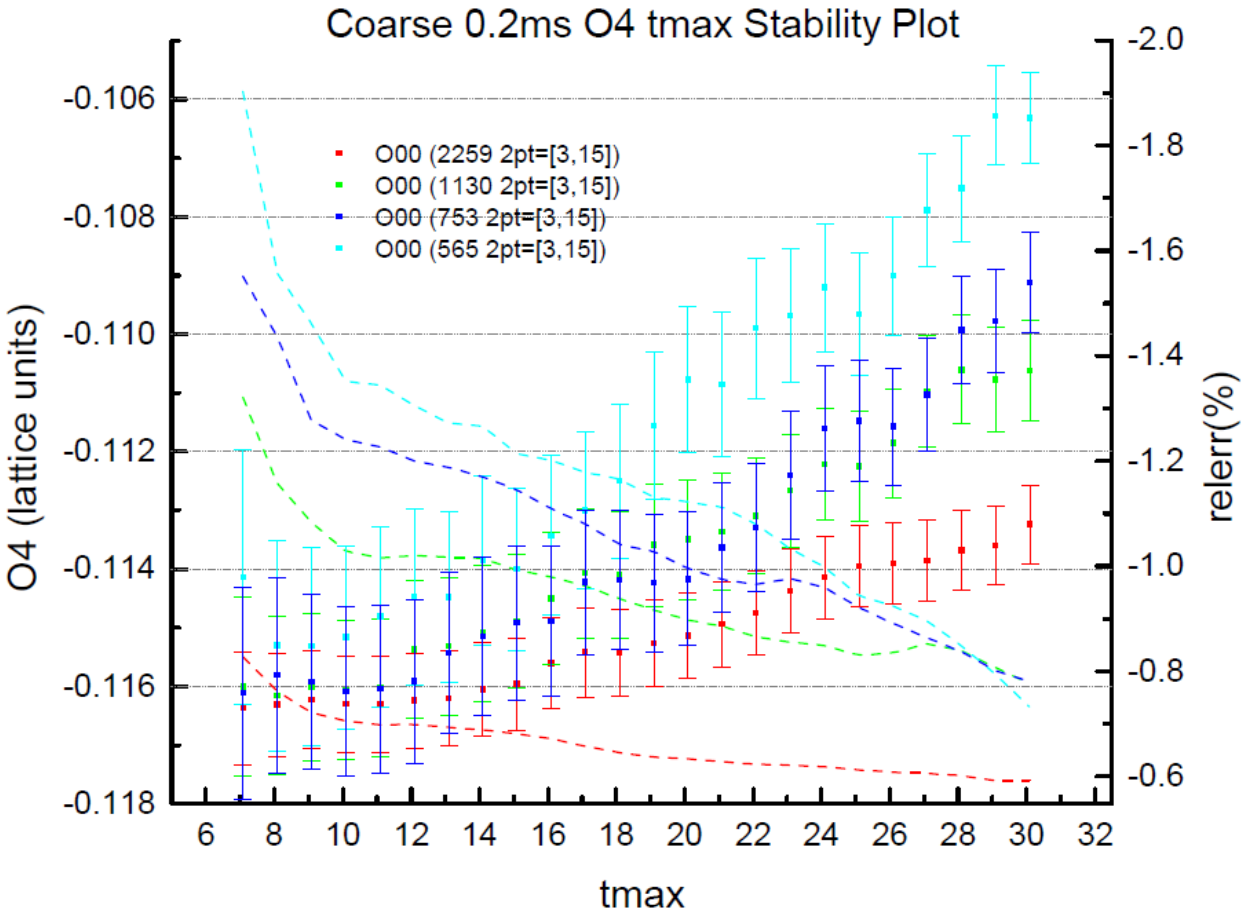
3-pt Tmax Stability Plot



What is wrong?

There is no stability. Fit drifts upwards in an unexpected way.

Stability vs. Configurations



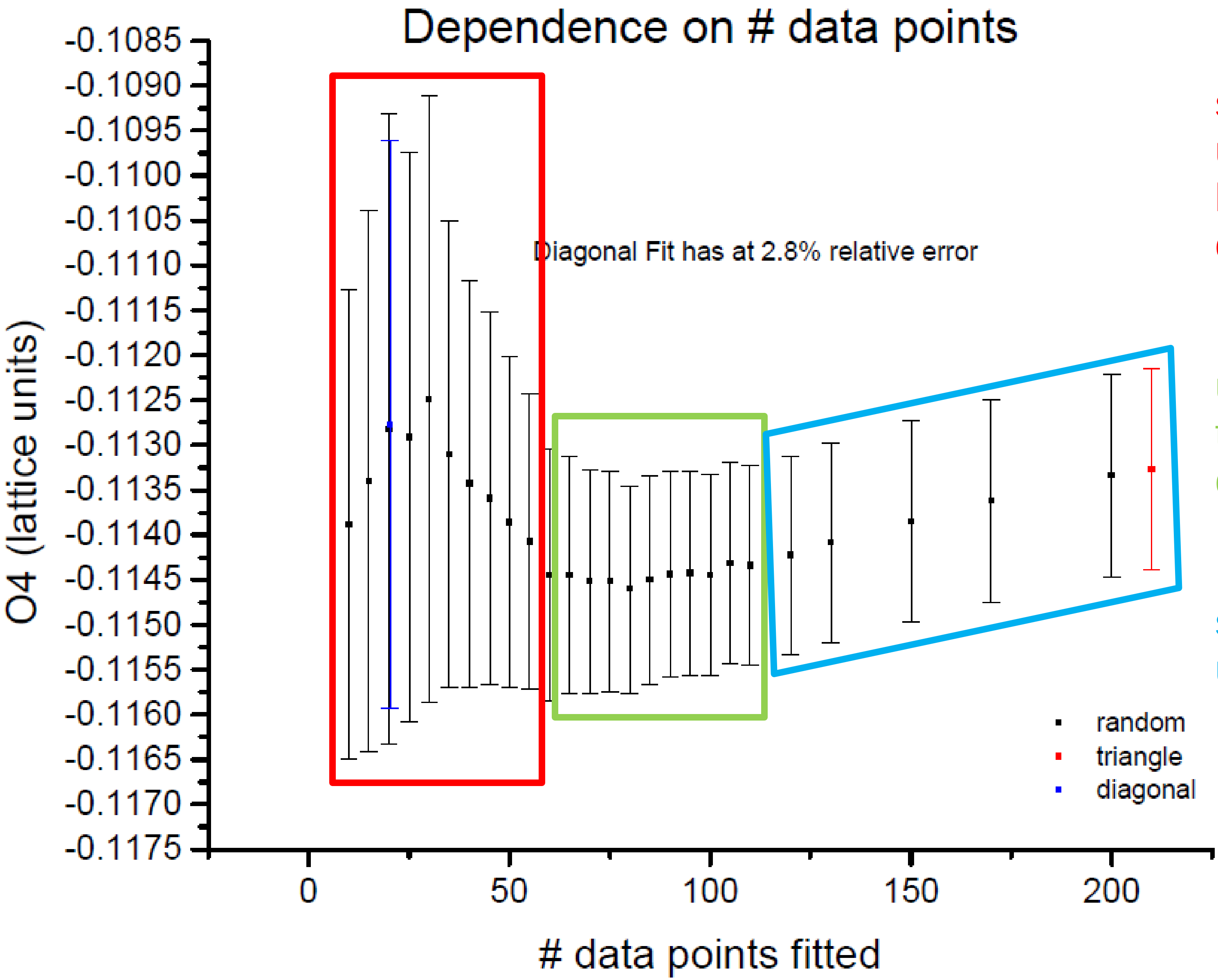
Summary of Plot

- These fits are over a square region. The trend in tmax is enhanced as I throw away configurations.
- Results are reproduced independently by running the same ensemble through Chris's code.

Issues with fitting towards large time

- Fitting a 2D slice up to large times will result in a covariance matrix with rank $\sim O(1000)$. Even for the coarse lattice with at most ~ 2000 configurations/ensemble, this is very dangerous.
- In depth analysis suggests the covariance matrix becomes ill-determined with inclusion of large amounts of data.

Stability vs. Data points



Region of high statistical uncertainty due to low number of data points.

Region of stability under change in the number of data points.

Region of high systematic uncertainty.

Covariance Matrix Regularization

Understanding this issue introduced and made me interested in the field of covariance matrix regularization.

This is a very current and interesting topic studied in the field of statistics.

Many papers talking about this issue and many proposals on different methods on regularization.

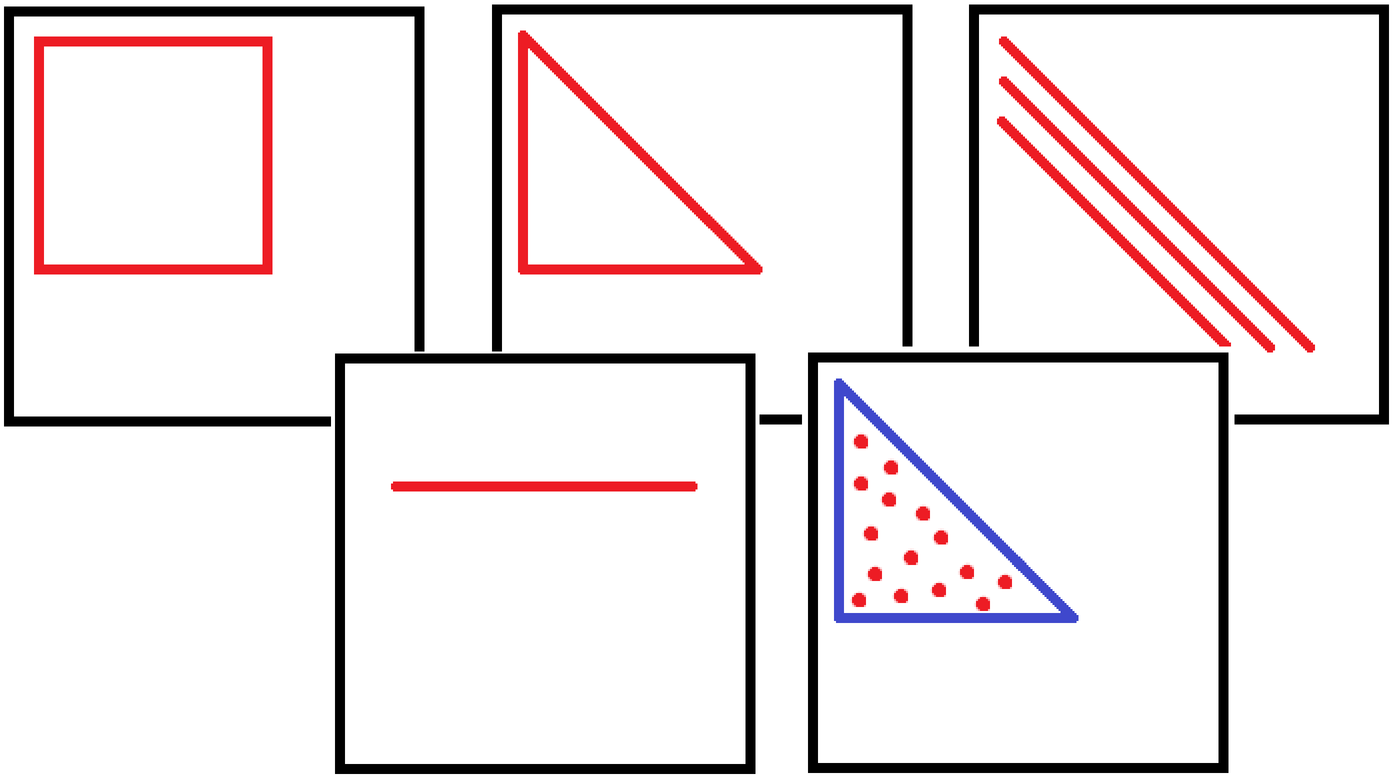
Goal is to control the size of the covariance matrix.

- Banding/Diagonal approximation
- Singular value decomposition cuts (Svdcut)

Challenges in Lattice QCD are multi-disciplinary!

Fit Regions

- Many different fit regions tried:
  - square, triangle, n-diagonal slices, horizontal slices, random sampling in triangle.



Main Point: Fitting along diagonal is the simplest choice.

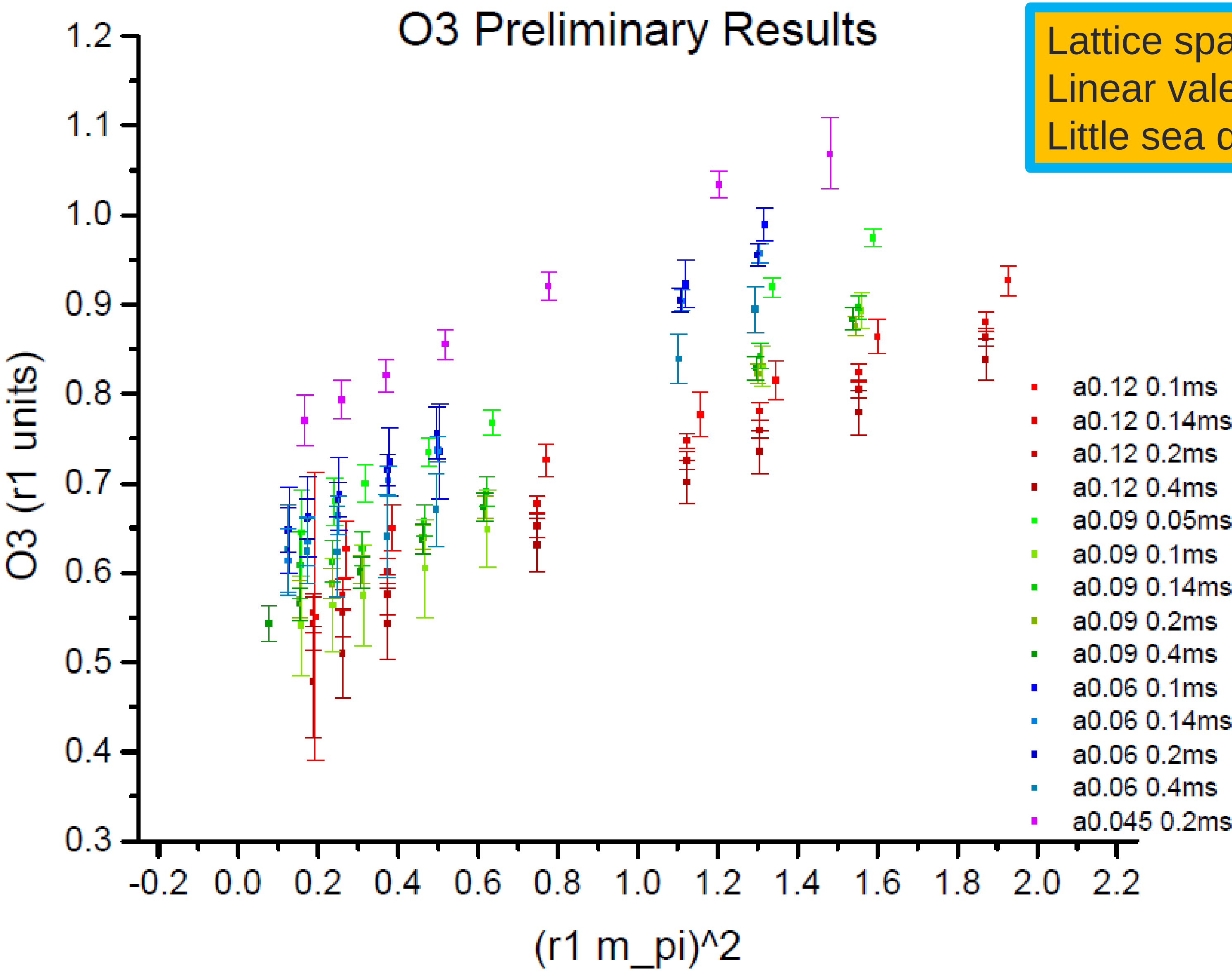
Renormalization and Matching

- One loop matching from Lattice effective theory to continuum QCD.
- Match Lattice regularization to dimensional regularization with the $\overline{MS}$ scheme.

$$\langle \mathcal{O}1 \rangle^{\overline{MS}} = [1 + \alpha_s \rho_{11}] \langle \mathcal{O}1 \rangle^{eff} + \alpha_s \rho_{12} \langle \mathcal{O}2 \rangle^{eff} + \mathcal{O}\left(\alpha_s^2, \alpha_s \Lambda_{QCD}/M\right)$$

- The matching coefficients are provided by Elvira Gamiz.

Preliminary Results



Lattice spacing dep.
Linear valence quark dep.
Little sea quark dep.

Future work

Kappa tuning corrections

Chiral and continuum extrapolation

Systematic error analysis

I would like to thank the Fermilab Fellowship of Theoretical Physics for funding my research here at Fermilab.

Extra Slides

Outline of Project

Current Progress

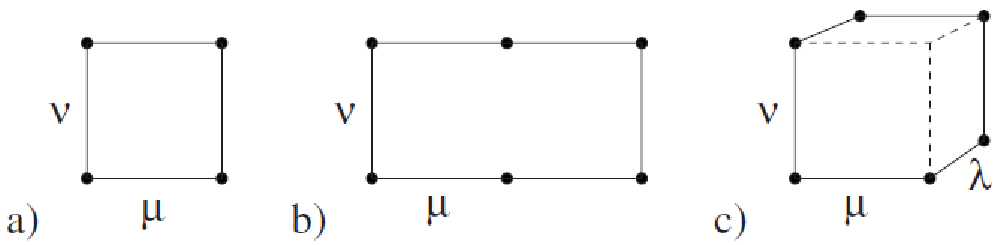
- Correlator Fits
  - Fit functions
  - Prior choices
  - Time range choices
  - Fit regions
  - Correlator fit results

Future To Do’s

- Kappa tuning corrections
- Chiral and Continuum extrapolation
- Systematic error analysis

Gluon Action

- Errors starting at order a^4



Wilson Loops:
a) Plaquette
b) 2x1 rectangle
c) 1x1x1 parallelogram
[Bazabov et al. MILC RMP 82, 1349 (2010)]

- Tadpole Improved
  - Decouple unphysical gluon loops

Order a^2 improv.

$$S_{LW} = \frac{\beta_{LW}}{3}\left\{\sum Re\,Tr(1-U_{pl}) - \sum \frac{1+0.4805\alpha_s}{20u_0^2} Re\,Tr(1-U_{rt}) - \sum \frac{0.03325\alpha_s}{u_0^2} Re\,Tr(1-U_{pg})\right\}$$

Computed by matching to physical quantities

Order $\alpha_s a^2$ improv.

- $\beta = 2N/g^2$
- $\beta_{LW} \equiv u_0^{-4}\beta c_{pl}$

Fermion Doubling

- The energy-momentum relation of the discretized Dirac action yields 16 degenerate fermions.

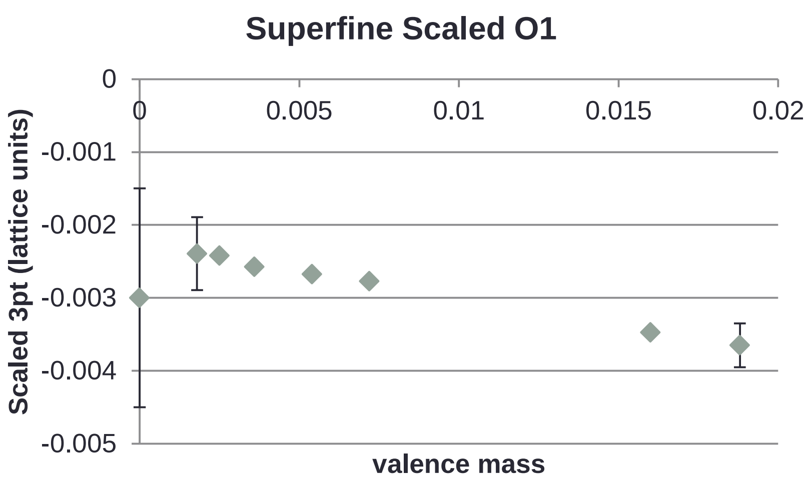
$$E^2 = m^2 + \sum_k \sin(q_k)$$

- For this project, two actions are used for the fermions.
  - Asqtad action for light quarks
  - Fermilab interpretation of the Clover action for heavy quarks.

Fit Procedure: Prior Choices

Consistent priors:
Same priors used for all ensembles with the same lattice spacing

- Ground state priors determined by scaled correlator plots. Example:

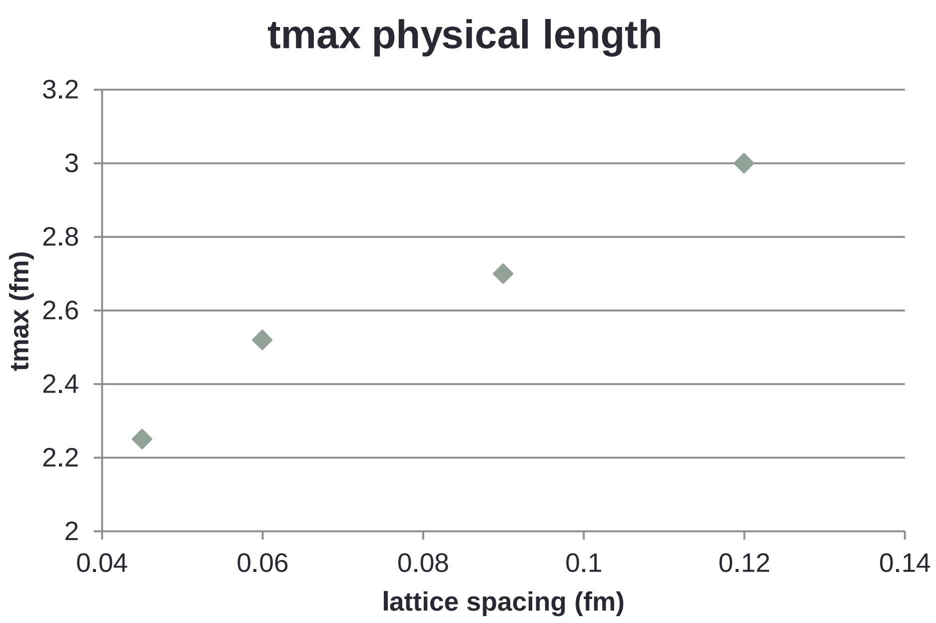


- Excited state energies determined by experimental or quark model predictions.
- Excited state amplitudes set to half the ground state due to operator smearing.
- Excited state matrix elements set to zero with width approx. 10x the ground state mean.

Following these guidelines yields consistent priors across lattice spacing

Fit Procedure: Time Range Choices

- Same time range for each lattice spacing.
- Tmin is physically the same distance away from the origin for all ensembles (0.72 fm).
- Tmax is varied smoothly across lattice spacing.

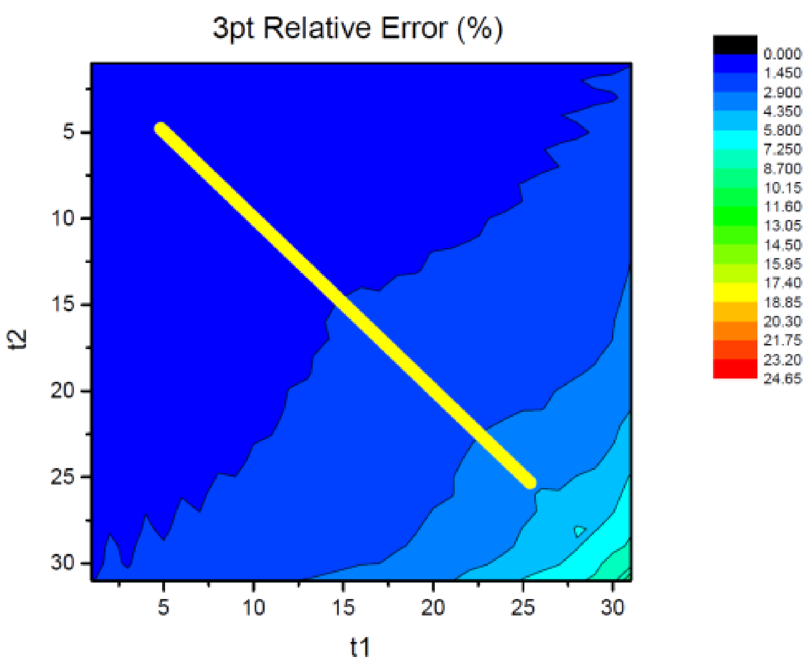


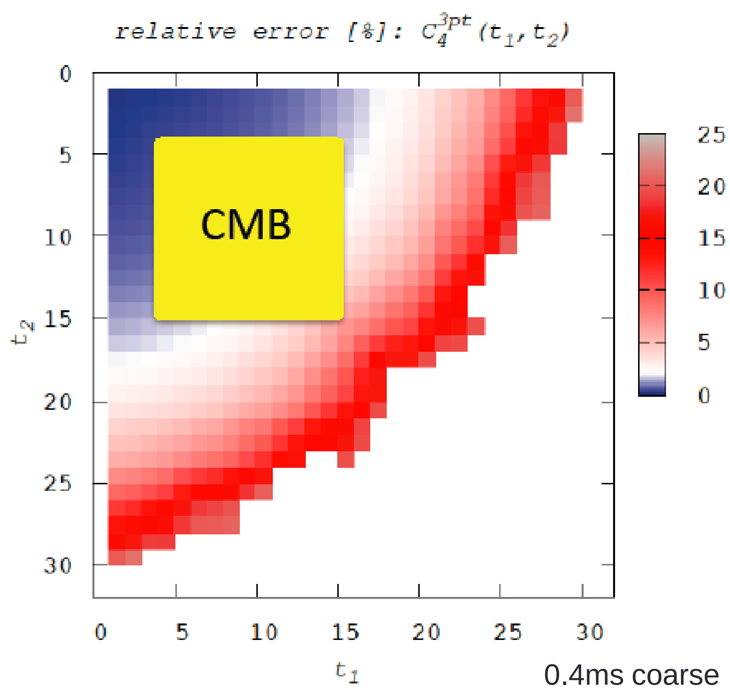
Notes:

- Time ranges are same for 2pt and 3pt fits.
- Varying Tmax is required to constrain rank of covariance matrix.
- These choices in are checked for stability.

Fit Procedure: Fit Region

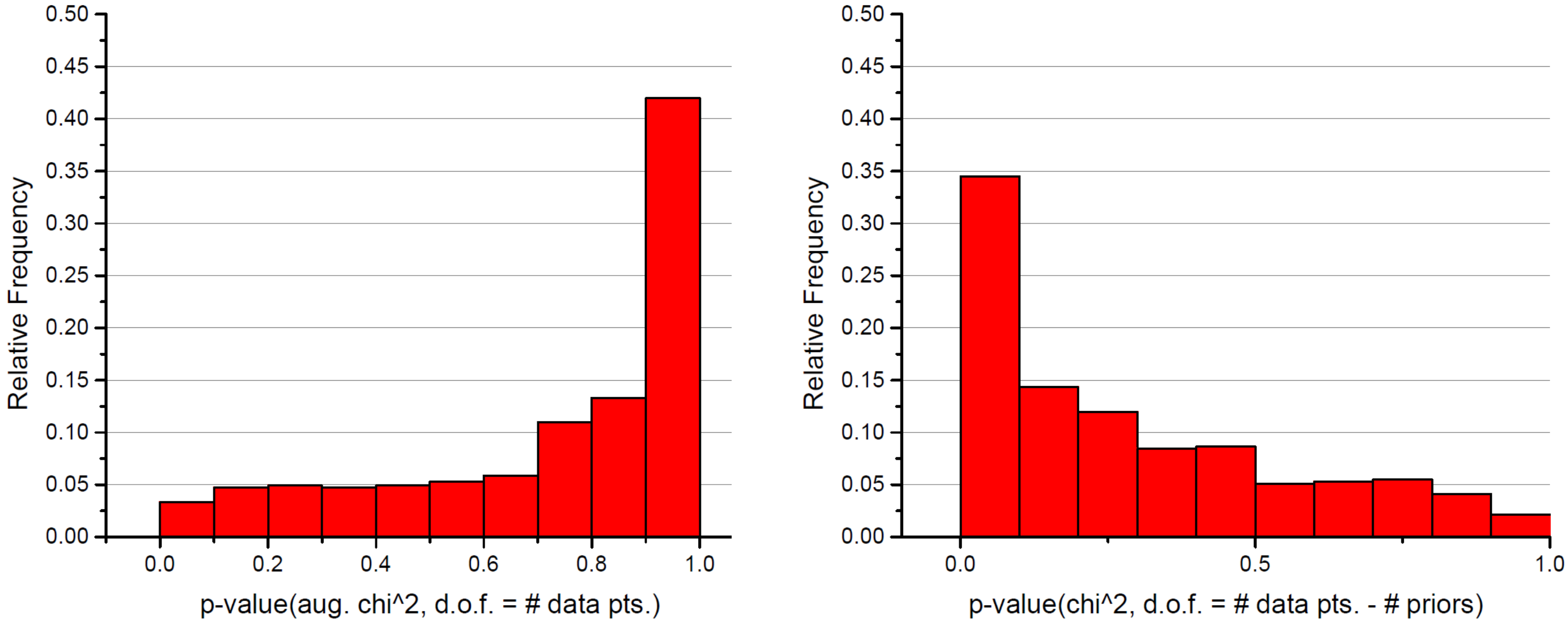
- The three point is fit along |t1|=t2.
- Different from B mixing where Chris fits a square and Elizabeth fits a fan shaped region.





0.4ms coarse

P-Value Distribution



- P-values distribution of all fits: determined by two ways.
- There is a big difference because the chi^2 distribution is shifted to the left by a lot when I subtract off the prior d.o.f. due to my fit region. (chi^2 ~ aug. chi^2)
- This p-value assumes the chi^2 is distributed as the integrand of the gamma function, which was shown to not be true by Ethan and Yuzhi. The correct degrees of freedom is also ambiguous when performing a constraint fit.

Fit shape discussion

- Square
  - Chris used this so I followed.
- Triangle
  - Data has symmetry across |t1|=t2. Fitting an nxn square yields n(n-1)/2 eigenvalues that are (numerically) equal to zero for the covariance matrix. This was postulated to cause issues, but loose svdcuts (1E-15) easily got rid of these modes and the fit results did not change.
- n-diagonal slices
  - Diagonal was the furthest away from PBCs. This was tried when PBC contributions were postulated to give contributions. Diagonal fits also significantly cut down the number of data points that were fitted. As I ramp up “n”, the n-diagonal fit degenerates to a triangle fit.
- Horizontal slices
  - Because it was easy to code.
- m-Random sampling in triangle
  - To convince myself that the upward trend seen in tmax (slide 12) was not due to PBCs, I sampled “m” points randomly inside a triangle. As I ramped up “m”, I reproduced the tmax trend. Therefore the trend is not a result of fitting closer to the PBC corners. (I restricted this to a triangle region so I don’t randomly sample the symmetric point across |t1|=t2).